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We estimate the Collins fragmentation function by introducing the effect of gluon rescattering in a model calculation of the fragmentation process. We include all necessary diagrams to the one-loop level and compute the nontrivial phases giving rise to the Collins function. We compare our results to the ones obtained from pion rescattering. We conclude that three out of four one-loop diagrams give sizeable contributions to the Collins function, and that the effect of gluon rescattering has a magnitude comparable to that of pion rescattering, but has opposite sign.

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## I. INTRODUCTION

The Collins fragmentation function [1] is an example of how the orientation of the quark spin can influence the direction of emission of hadrons in the fragmentation process. The existence and the features of this function are related to important questions of nonperturbative QCD, such as the role of chiral symmetry and color gauge invariance in the hadronization process. The Collins function is believed to be at the origin of single-spin asymmetries in hard hadronic reactions [2, 3, 4, 5, 6, 7], which lead to attempts of estimating it from phenomenology [8, 9, 10, 11, 12].

The Collins function is a so-called T-odd entity. T-odd functions typically require the interference between two amplitudes with different imaginary parts to exist. Perturbative calculations of fragmentation functions in quantum field theories can provide – through loop corrections – the necessary nontrivial phases in the fragmentation amplitude [13].

The Collins fragmentation function has been estimated in a chiral invariant approach where the effective degrees of freedom are constituent quarks and pions, coupled via a pseudovector interaction [14]. In order to generate the required phases, one-loop corrections to the quark propagator and vertex have been included.

In the meanwhile, gluon loop corrections in distribution functions have been investigated. In the context of a spectator model of the nucleon, it has been shown that the exchange of a gluon between the struck quark and the target spectators gives rise to T-odd distribution functions [15, 16]. This has been interpreted as the one-gluon approximation to the gauge link, which is included in the definition of parton distribution functions and insures their color gauge invariance [16, 17, 18, 19]. It was soon realized that gauge link effects have different signs in semi-inclusive DIS and Drell-Yan scattering [16, 20], implying different signs for T-odd distribution functions in the two processes. This is for the first time an example of violation of universality of the distribution functions, though it consists simply of a sign change. Gluon rescattering has been used thereafter to estimate T-odd distribution functions [21, 22].

As in the distribution functions, the gauge link can generate nontrivial phases and T-odd effects in the fragmentation functions as well. This was analyzed in Ref. [23], where it was shown in particular that one-gluon contributions to the gauge link don't change sign in semi-inclusive DIS and in  $e^+e^-$  annihilation.

In this work, we model the tree-level fragmentation process in the same way as in Ref. [14]. To generate T-odd effects, instead of introducing pion-loop corrections, we include gluon rescattering corrections. We take into consideration all possible one-loop amplitudes, including those that arise from the gauge link. We compute the Collins function and its first two moments, which can be experimentally accessed in single-spin asymmetries in semi-inclusive DIS, as well as in  $e^+e^-$  annihilation. A similar calculation has been presented very recently by Gamberg et al. [24], using a pseudoscalar coupling complemented with a Gaussian form factor to model the tree level fragmentation. However, in that work only one of the four possible gluon rescattering contributions has been taken into account.

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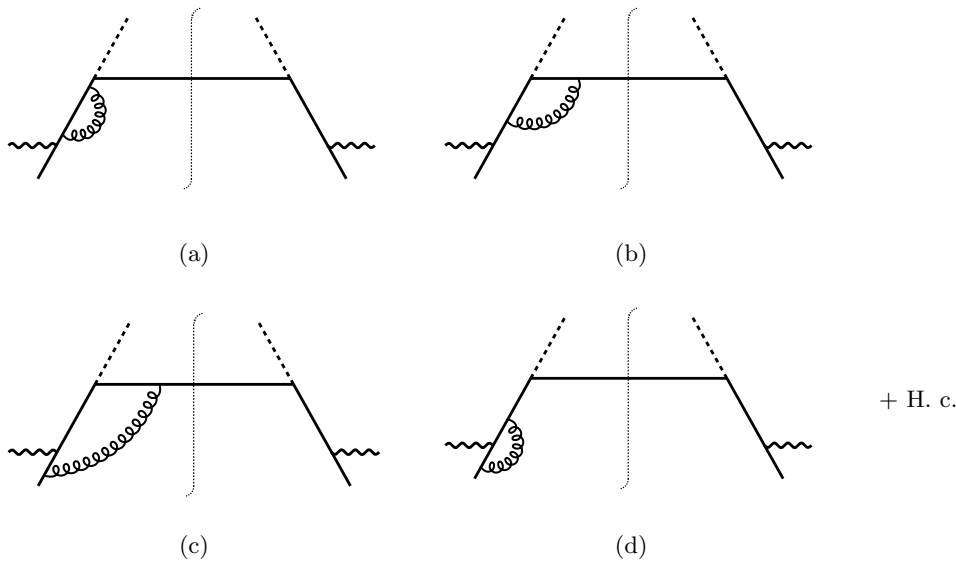


FIG. 1: Single gluon-loop corrections to the fragmentation of a quark into a pion contributing to the Collins function. The Hermitian conjugate diagrams (H. c.) are not shown explicitly.

## II. CALCULATION OF THE COLLINS FUNCTION

We use the following definition of the Collins function  $H_1^\perp$  [25, 26]:

$$\frac{\epsilon_T^{ij} k_{Tj}}{m_\pi} H_1^\perp(z, z^2 k_T^2) = \text{Tr} [\Delta(z, k_T) i\sigma^{i-} \gamma_5]. \quad (1)$$

The correlation function  $\Delta(z, k_T)$  can be written as [19]

$$\begin{aligned} \Delta(z, k_T) &= \frac{1}{4z} \int dk^+ \Delta(k, p) \Big|_{k^- = p^- / z} \\ &= \sum_X \int \frac{d\xi^+ d^2\xi_T}{4z(2\pi)^3} e^{+ik \cdot \xi} \langle 0 | \mathcal{L}_{[-\infty^+, \xi^+]} \psi(\xi) | \pi, X \rangle \langle \pi, X | \bar{\psi}(0) \mathcal{L}_{[0^+, -\infty^+]} | 0 \rangle \Big|_{\xi^- = 0}, \end{aligned} \quad (2)$$

where the notation  $\mathcal{L}_{[a,b]}$  indicates a straight gauge link running from  $a$  to  $b$ . In this work, we will use the Feynman gauge. In the case of transverse-momentum independent fragmentation functions, i.e., after integrating over  $k_T$ , by choosing a light-cone gauge the link can be reduced to unity. However, in the case of transverse-momentum dependent functions – as the Collins function – the gauge link cannot be neglected and becomes one of the possible sources of nontrivial phases in the fragmentation amplitude and thus of T-odd fragmentation functions [23]. Note that in case of transverse-momentum dependent functions with lightlike Wilson lines, divergencies can appear [27, 28]. However, for our study this problem does not show up.

To model the tree-level fragmentation process, we make use of the chiral invariant effective model of Manohar and Georgi [29]. The unpolarized fragmentation function in this model reads [14]

$$D_1(z, z^2 k_T^2) = \frac{1}{z} \frac{g_A^2}{4F_\pi^2} \frac{1}{16\pi^3} \left( 1 - 4 \frac{1-z}{z^2} \frac{m^2 m_\pi^2}{[k_T^2 + m^2 + m_\pi^2(1-z)/z^2]^2} \right). \quad (3)$$

To obtain a nonzero Collins function, we have to compute one-loop contributions. In contrast to what was done in our previous calculation [14], instead of introducing pion loops, we now consider the effect of including gluon rescattering.

In Fig. 1 we illustrate the Feynman diagrams involved in the calculation of the Collins function. Diagrams (a) and (b) – the self-energy and pion vertex corrections – have analogous contributions in the pion-loop case. However, due to gauge invariance we have to take into consideration also diagrams (c) and (d), which we will henceforth call the box and photon vertex corrections. Despite their appearance, these contributions do not break factorization, since they

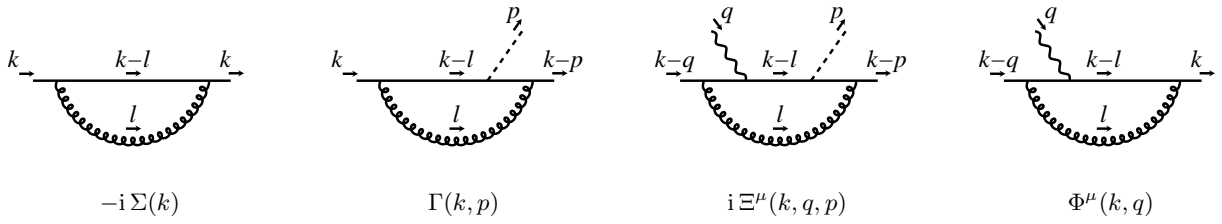


FIG. 2: One-loop self-energy, pion vertex, box, and photon vertex corrections.

can be interpreted as contributions to the gauge link, which is included in Eq. (2) [16, 17, 18, 19]. In fact, diagram (d) contributes not only to the gauge link of the fragmentation function. However, since in this specific calculation its contribution turns out to vanish, we will avoid considering this issue.

In all these diagrams, the gluon loops can give rise to nontrivial phases to the scattering amplitude. Through the interference with the tree-level amplitude, these contributions generate nonzero T-odd fragmentation functions, such as the Collins function. The portions of the diagrams relevant for the generation of imaginary parts are sketched in Fig. 2. They can be expressed analytically as

$$\Sigma(k) = -ig_s^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\gamma_\rho (\not{k} - \not{l} + m) \gamma^\rho}{[(k-l)^2 - m^2][l^2 - m_g^2]}, \quad (4)$$

$$\Gamma(k, p) = -i \frac{g_A}{2F_\pi} g_s^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\gamma_\rho (\not{k} - \not{p} - \not{l} + m) \not{p} \gamma_5 (\not{k} - \not{l} + m) \gamma^\rho}{[(k-p-l)^2 - m^2][(k-l)^2 - m^2][l^2 - m_g^2]}, \quad (5)$$

$$\Xi^\mu(k, q, p) = -i \frac{g_A}{2F_\pi} g_s^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\gamma_\rho (\not{k} - \not{p} - \not{l} + m) \not{p} \gamma_5}{[(k-p-l)^2 - m^2][l^2 - m_g^2]} \frac{(\not{k} - \not{l} + m) \gamma^\mu (\not{k} - \not{q} - \not{l} + m) \gamma^\rho}{[(k-l)^2 - m^2][(k-q-l)^2 - m^2]}, \quad (6)$$

$$\Phi^\mu(k, q) = -ig_s^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\gamma_\rho (\not{k} - \not{l} + m) \gamma^\mu}{[(k-q-l)^2 - m^2]} \frac{(\not{k} - \not{q} - \not{l} + m) \gamma^\rho}{[(k-l)^2 - m^2][l^2 - m_g^2]}, \quad (7)$$

where  $C_F = 4/3$ . We choose the kinematics in the following way (in light-cone coordinates):

$$q = \left[ \frac{Q}{\sqrt{2}}, -\frac{Q}{\sqrt{2}}, 0_T \right], \quad k = \left[ \frac{Q}{\sqrt{2}}, \frac{k^2 + k_T^2}{Q\sqrt{2}}, k_T \right], \quad p = \left[ z \frac{Q}{\sqrt{2}}, \frac{m_\pi^2}{zQ\sqrt{2}}, 0_T \right]. \quad (8)$$

To identify the contributions to the Collins function, we write down the explicit expressions for the cut diagrams of Fig. 1 and we apply the definition of the Collins function given in Eq. (1).

It turns out that only some specific elements of the imaginary parts of the diagrams in Fig. 2 contribute to the Collins function. For simplicity we will denote them as  $\text{Im } \sigma$ ,  $\text{Im } \gamma$ ,  $\text{Im } \xi$ ,  $\text{Im } \phi$ . The formula of the Collins function can be written then as

$$\begin{aligned} H_1^\perp(z, z^2 k_T^2) &= \frac{g_A^2 m_\pi}{16\pi^3 F_\pi^2} m k^- \int dk^+ \delta((k-p)^2 - m^2) \frac{1}{k^2 - m^2} (\text{Im } \sigma + \text{Im } \gamma + \text{Im } \xi + \text{Im } \phi) \\ &= \frac{g_A^2}{32\pi^3 F_\pi^2} \frac{m_\pi}{1-z} \frac{m}{k^2 - m^2} (\text{Im } \sigma + \text{Im } \gamma + \text{Im } \xi + \text{Im } \phi) \Big|_{k^2 = k_T^2 \frac{z}{1-z} + \frac{m^2}{1-z} + \frac{m_\pi^2}{z}}. \end{aligned} \quad (9)$$

Considering only the leading power in  $Q$  and assuming the gluons to be massless, the calculation of the four diagrams yields

$$\text{Im } \sigma = -\frac{\alpha_s}{2\pi} C_F \left( 3 - \frac{m^2}{k^2} \right) I_{1,g}, \quad (10)$$

$$\text{Im } \gamma = \frac{\alpha_s}{2\pi} C_F \left[ \left( 1 + \frac{m^2}{k^2} \right) I_{1,g} + 4m_\pi^2 I_{2,g} \right], \quad (11)$$

$$\begin{aligned} \text{Im } \xi &= -\frac{\alpha_s}{\pi} C_F \left\{ 2 I_{1,g} + 2Q^2 \left( (k^2 - m^2)(1-z) I_{4,g} + I_{3,g} \right) + 2 \left( 2zm^2 - (1-z)(k^2 - m^2) \right) \right. \\ &\quad \times \frac{1}{2z^2 k_T^2} \left[ z Q^2 \left( (k^2 - m^2)(1-z) I_{4,g} + I_{3,g} \right) - \left( z(k^2 - m^2 + m_\pi^2) - 2m_\pi^2 \right) I_{2,g} \right] \Big\}, \end{aligned} \quad (12)$$

$$\text{Im } \phi = 0, \quad (13)$$

where the integrals introduced above correspond to

$$I_{1,g} = \int d^4l \delta(l^2) \delta((k-l)^2 - m^2) = \frac{\pi}{2k^2} (k^2 - m^2) \theta(k^2 - m^2), \quad (14)$$

$$I_{2,g} = \int d^4l \frac{\delta(l^2) \delta((k-l)^2 - m^2)}{(k-p-l)^2 - m^2} = -\frac{\pi}{2\sqrt{\lambda(k^2, m^2, m_\pi^2)}} \times \ln \left( 1 + \frac{2\sqrt{\lambda(k^2, m^2, m_\pi^2)}}{k^2 - m_\pi^2 + m^2 - \sqrt{\lambda(k^2, m^2, m_\pi^2)}} \right) \theta(k^2 - m^2), \quad (15)$$

$$I_{3,g} = \int d^4l \frac{\delta(l^2) \delta((k-l)^2 - m^2)}{(k-q-l)^2 - m^2} = -\frac{\pi}{Q^2} \ln \frac{Q^2}{m\sqrt{k^2}} \theta(k^2 - m^2), \quad (16)$$

$$I_{4,g} = \int d^4l \frac{\delta(l^2) \delta((k-l)^2 - m^2)}{[(k-p-l)^2 - m^2][(k-q-l)^2 - m^2]} = \frac{\pi}{Q^2} \frac{1}{(k^2 - m^2)(1-z)} \left( \ln \frac{Q^2}{m\sqrt{k^2}} + \ln \frac{\sqrt{k^2}(1-z)}{m} \right) \theta(k^2 - m^2). \quad (17)$$

In the previous formulae, we made use of the Källén function,  $\lambda(k^2, m^2, m_\pi^2) = [k^2 - (m + m_\pi)^2][k^2 - (m - m_\pi)^2]$ . Note that the final result for the Collins function is independent of  $Q^2$ .

### III. NUMERICAL RESULTS

We perform the numerical integration over the transverse momentum according to

$$D_1(z) = \pi \int_0^{K_T^2 \text{max}} dK_T^2 D_1(z, K_T^2), \quad (18)$$

where  $K_T = -zk_T$ . We impose a cutoff on the virtuality of the fragmenting quark, so that  $k^2 \leq \mu^2$ . Due to the kinematics, this choice fixes the upper limit of the  $K_T^2$  integration to

$$K_T^2 \text{max} = z(1-z)\mu^2 - zm^2 - (1-z)m_\pi^2. \quad (19)$$

For the numerical computations, we use the following values of the parameters of our model:

$$m_q = 0.3 \text{ GeV}, \quad g_A = 1, \quad \mu^2 = 1 \text{ GeV}^2, \quad \alpha_s = 0.3. \quad (20)$$

The choice of the first three parameters has been extensively discussed in Ref. [14]. The choice of the strong coupling constant  $\alpha_s$  corresponds to a reasonable value for  $Q^2 \approx 1 \text{ GeV}^2$ .

In Fig. 3 and 4 we plot the ratio

$$\frac{H_1^{\perp(1/2)}(z)}{D_1(z)} \equiv \frac{\pi}{D_1(z)} \int dK_T^2 \frac{|K_T|}{2zm_\pi} H_1^\perp(z, K_T^2), \quad (21)$$

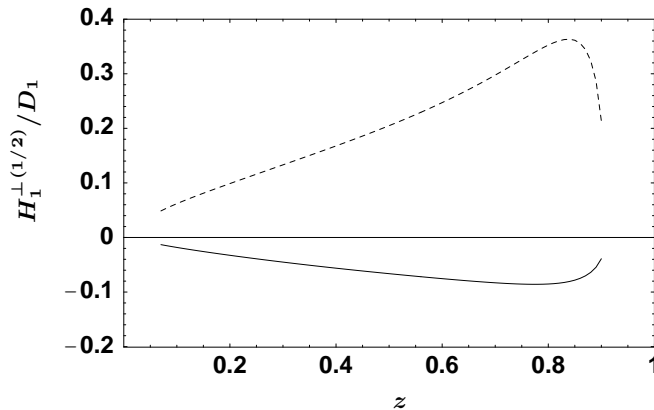


FIG. 3: Result for  $H_1^{\perp(1/2)}/D_1$  including only the self-energy and vertex gluon-loop corrections (solid line) and comparison with the result of the pion-loop model (dashed line) from Ref. [14].

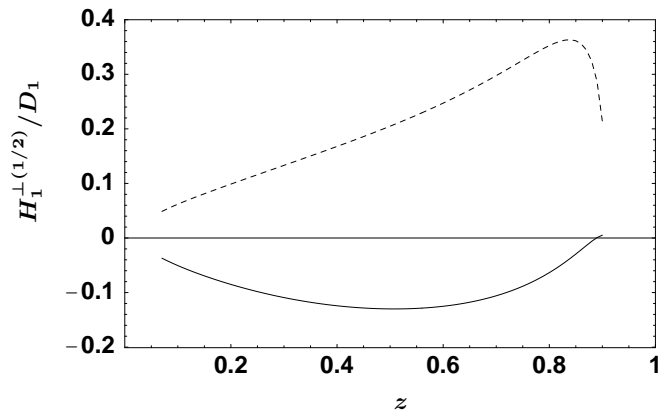


FIG. 4: Result for  $H_1^{\perp(1/2)}/D_1$  including all gluon-loop contributions (solid line) and comparison with the result of the pion-loop model (dashed line) from Ref. [14].

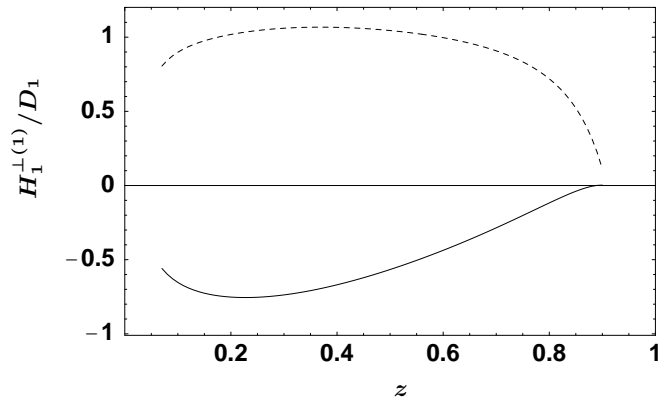


FIG. 5: Result for  $H_1^{\perp(1)}/D_1$  including all gluon-loop contributions (solid line) and comparison with the result of the pion-loop model (dashed line) from Ref. [14].

which enters the unweighted transverse single spin asymmetries for pion production in semi-inclusive DIS [14, 26]. In Fig. 3 we show only the contributions of the self-energy and pion vertex corrections, diagrams (a) and (b) of Fig. 1. They have a direct correspondence to the pion-loop case. As shown by the plot, these contributions are smaller and have an opposite sign compared to the pion-loop ones. In Fig. 3 we show the sum of all gluon-loop contributions, i.e. all diagrams of Fig. 1, including in particular the box diagram, which contributes to the gauge link and has no analogous term in the pion case.

In Fig. 5 we plot the ratio

$$\frac{H_1^{\perp(1)}(z)}{D_1(z)} \equiv \frac{\pi}{D_1(z)} \int dK_T^2 \frac{K_T^2}{2z^2 m_\pi^2} H_1^\perp(z, K_T^2), \quad (22)$$

which typically enters weighted transverse spin asymmetries in semi-inclusive DIS [14, 30, 31].

#### IV. CONCLUSIONS

In this work we have calculated the Collins function for pions describing the fragmentation process at tree level by means of a chiral invariant effective theory, and generating the required nontrivial phases by means of gluon rescattering. We have computed all necessary diagrams contributing to the Collins function at the one-loop level, including the ones involved in the gauge link. Out of the four diagrams we considered, one gives no contribution, while the other three have similar magnitudes and cannot be neglected. A word of caution is in order at this point: we have performed a perturbative expansion in  $\alpha_S$  in a regime where its justification can be questioned. Moreover, we emphasize once again that this is only a model calculation, whose reliability might be very limited. Nevertheless, we believe that it could give an indication of the influence of the gluon dynamics on the Collins function.

We compared our results with those formerly obtained in Ref. [14], where pion rescattering was considered as a possible source of nontrivial phases. The two approaches are identical at tree level, allowing a clear comparison of the

differences at the one-loop level. We have found that the gluon rescattering mechanism produces a Collins function with opposite sign compared to the pion rescattering mechanism. The two effects are similar in magnitude, except perhaps at high  $z$ , where pion rescattering dominates. Cancellations between these two competing mechanism could decrease the experimental asymmetries.

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